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DISCRIMINATION AND DEPLOYMENT REQUIREMENTS FOR LAYERED DEFENSE --ETC(U)
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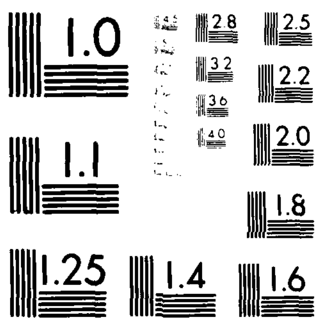

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LINCOLN LABORATORY

DISCRIMINATION AND DEPLOYMENT REQUIREMENTS FOR LAYERED DEFENSE SYSTEMS

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Group 32

TECHNICAL REPORT 555

18 FEBRUARY 1981

Approved for public release; distribution unlimited.

LEXINGTON MASSACHUSETTS

ABSTRACT

A mathematical model is described which permits an analysis of optimal attack and deployment of a layered defense system. The model can also be used to determine discrimination requirements of such a system, and optimal RV/decoy exchange ratios.

CONTENTS

I. INTRODUCTION

For several years the use of layered defense systems to protect MM or MX missile fields has been of interest to the ballistic missile defense (BMD) community. Such systems provide complementary operation of two defensive layers, and allow benefits deriving from the use of adaptive-preferential defense, multiple non-nuclear exo-atmospheric kill vehicles, and generally low leakage.

In the following pages we will describe the construction and usage of a mathematical model of a layered defense system (LDS). In general, this model allows us to observe the operational trade-offs inherent in any LDS. More precisely, it is assumed that the Offense has the option of deploying two types of decoys in addition to nuclear warheads (RVs), up to a given throw-weight level. One type of decoy is designed to be deployed at high altitudes (exo-atmospheric) and to confuse the overlay component of the LDS. The other type is designed to operate at low altitude (endo-atmospheric) and to confuse the underlay component of the LDS. The basic output of our model is the precise targeting pattern for the Offense which serves to minimize the overall survival probability of the Defense's targets. Such an attack pattern is said to be "optimal".

These optimal attack patterns and corresponding target survival probabilities depend on several parameters. In addition to those which describe the deployment quantities of the LDS (particularly: number of interceptors), the Defense's discrimination capabilities (K-factors) in each layer, the Offense's off-loading capabilities (RV/decoy exchange ratios), and the various interceptor/warhead reliabilities are also of interest and importance. Our model permits sensitivity studies to determine the effects of such parameters on system operation. In turn these studies allow us to infer such information as discrimination requirements and optimal RV/decoy exchange ratios.

In the next three sections we will precisely formulate the problems to be considered and briefly and schematically review LDS operation. We will then describe the basic model and in several subsequent sections illustrate prototypical model output and the kinds of answers it provides to the indicated problems.

II. PROBLEM STATEMENT

It is, of course, important to carefully state the assumptions underlying our problem formulation, as they will determine the eventual output and conclusions from the ensuing model.

The Defense has a fixed collection of targets which it elects to protect by means of a layered defense. These targets are of uniform value to the Defense. (The case of targets not of uniform value constitutes an *interesting excursion*, but will not be further considered at this time.) The overlay (OL) layer consists of interceptors, more precisely, multiple kill vehicles (MKVs), supported by the necessary complement of probes, busses, and battle managers. Each interceptor is of uniform reliability characterized by a leakage OLK, defined as the probability that the interceptor fails to destroy a given attacking object at which it is aimed. This OL layer serves to defend the entire target complex.

There is also an underlay (UL) component of the defensive system, consisting of a few (possibly none) short range, high acceleration interceptors defending each target, along with supporting sensors and data processors. These interceptors are also of uniform reliability, as characterized by a leakage probability ULK. There is no mutual defense capability assumed here, so that each interceptor can defend only a single target. Further, the Offense is assumed to know how the UL interceptors are deployed (non-deceptive deployment).

On the other side, the Offense is constrained by a throw-weight bound or threat level TL. This is the total throw-weight as measured in RVs divided by the total number of targets. The Offense also has the capability of off-loading RVs for either of two types of decoys. It is assumed that one type of decoy is designed to be effective against high altitude (optical) sensors, and the other type against low altitude sensors (radars). We refer to the first type as an OL decoy and the second type as a UL decoy.

The offensive RVs are assumed to be of uniform reliability. If not successfully intercepted by the Defense, each RV has a probability PK (deduced from accuracy and warhead reliability estimates) of destroying its target.

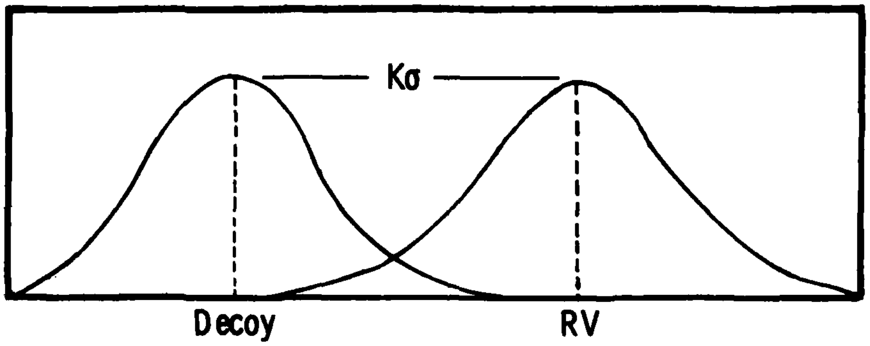
Finally, the Defense is assumed to employ, in each layer, various sensors which supply discrimination information about approaching targets. This information consists of the value of some observable attribute of a target. (The actual choice of such discriminants is still a controversial matter, but for present purposes this choice is immaterial.) It is assumed that the larger the observable the more likely the target is to be an RV and the measurements of the observable from RV and from decoy obey Gaussian probability laws of equal variance. This situation is depicted in Fig. 1, and it permits us to encapsulate the Defense's

Based on earlier (1977) work on (single layer) terminal defense by Dunn, Chang, Rheinstein

Each layer utilizes one (or several) sensors which supply discrimination information by evaluating some observable attribute of enemy objects

Observables assumed normally distributed

- Equal variances (σ^2)
- Means separated by $K\sigma$ units
- Two values of K: K_{OL}, K_{UL}



Density Functions of a Particular RV/Decoy Observable

Fig. 1. Defense discrimination model.

discrimination capability into two numbers K_{OL} and K_{UL}. That is, K_{OL} is the difference between the means of the RV and OL decoy observables, normalized by the common standard deviation; K_{UL} is the analogous measure for the UL layer.

In general, K-values in excess of 4 are representative of very good discrimination capability; by contrast, a perfect decoy (of either type) results in a K-value of zero.

Given all this hypothetical Offense-Defense capability, the basic questions that occur pertain to the nature of optimal deployments by each side, and specifically to the discrimination requirements of the Defense. We shall formulate these and related questions more precisely, after first reviewing briefly the operation of a prototypical LDS, and indicating a few important hypotheses about such operation.

III. LDS OPERATION

In Fig. 2 we display a flowchart which schematically illustrates the operation of any LDS, independently of system specifics. Initially, the Defense observes (via satellites, probes, etc.) an incoming swarm of RVs and decoys. After appropriate detection, designation and discrimination (D^3) and command, control and communication (C^3) activities, there is an encounter between these attackers and the OL layer of defense. As a result some RVs are destroyed; the remainder, along with the UL decoys, continue on course. This RV leakage results for several reasons which we broadly designate as system leakage and sensor leakage. Subsequently there is a second and final encounter with the UL layer of defense. For the same general reasons some RVs may penetrate this layer too; those that do destroy their assigned targets with probability PK

We now discuss in greater detail the leakages just designated. By system leakage we mean either interceptor malfunction (with associated probabilities OLK, ULK already assigned), interceptor exhaustion, or, in the OL component, inadequacies of battle management. This latter problem implies that the Defense is unable to launch an interceptor at an incoming object which it would otherwise wish to destroy.

As for sensor leakage there are, in general, several possible outcomes when the Defense processes sensor data to determine the identity of an incoming object. There may first be a failure to

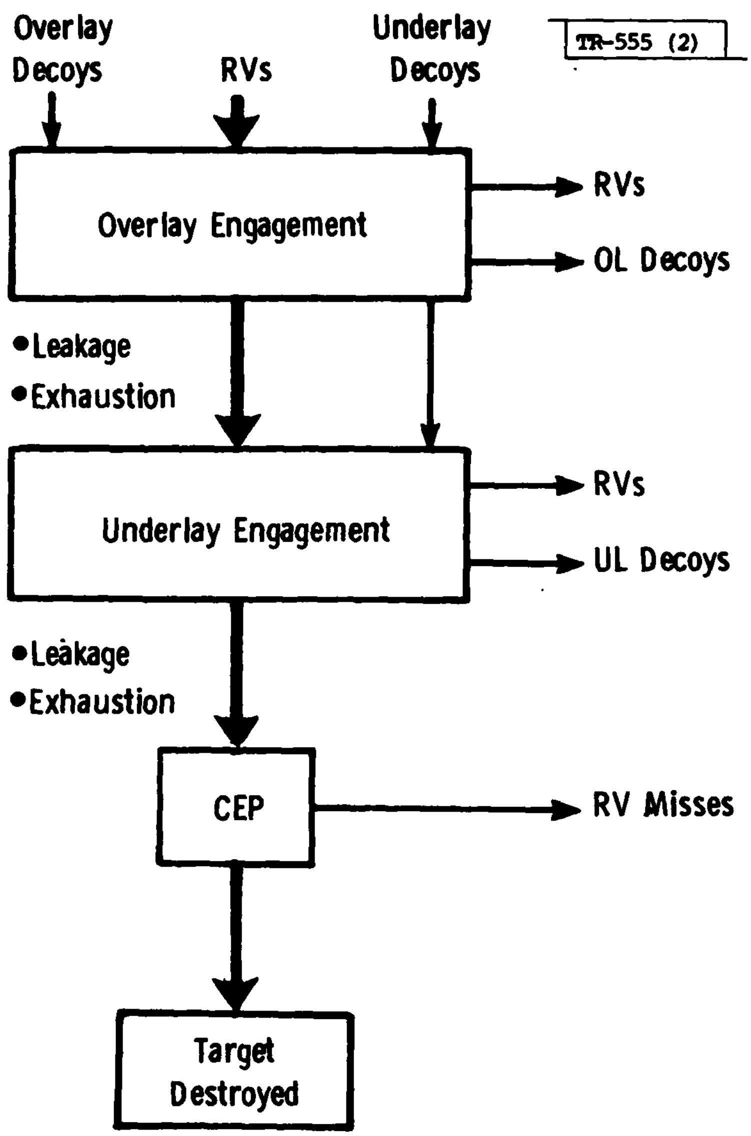


Fig. 2. Flowchart for LDS operation.

acquire and/or track the object. When the object is in fact tracked there are four further possibilities. Two of them are correct classification (RV = RV, decoy = decoy), and two are incorrect: (RV = decoy, decoy = RV). This latter misclassification is commonly termed a "false alarm"; we will call the first misclassification discrimination leakage, although it is often also just called sensor leakage. We let P_{DL} and P_{FA} denote the probabilities of these two types of misclassification. Under our assumed discrimination model these probabilities do not fluctuate independently but are functionally related: for each fixed value of K (in either layer) a value of P_{DL} determines a value of the observable and that, in turn, specifies P_{FA}. The corresponding curves in the (P_{DL}, P_{FA}) - plane, indexed by K, are known as operating characteristics.

To carry out the OL engagement there are, roughly speaking, three types of discrimination logic that the Defense might employ. The least favorable is to simply set an observable threshold, thereby determining an operating point (P_{DL}, P_{FA}), and launch an interceptor at any object whose observable measurement exceeds the specified threshold. Such a logic is called "defense conservative". At the other extreme, the Defense may be able to perceive and evaluate the entire threat before committing any interceptors. This of course involves a more sophisticated sensor and battle management capability. If so, the incoming objects can be ranked by their observable values, and interceptors launched only at the highest ranked (hence presumably most threatening) objects, so that the defense always uses the appropriate number of interceptors. This logic is called "offense conservative". The threshold which determines the launch commitment is thus implicitly set by the result of the sensor information, rather than being specified a priori Finally, there is a spectrum of possible assignment logics between these extremes, which basically involve a variable threshold; this is changed by the Defense as the attack evolves, based on analysis of partial information of the threat.

We now indicate several hypotheses about the operation of the LDS which remain in force throughout this report. These hypotheses are very important as they strongly influence the eventual structure and output of our optimization model.

1) Both layers of defense operate in offense conservative mode.

2) The OL layer operates subtractively. That is, each incoming object believed to be an RV is treated as equally threatening in that the Defense does not know how it is targeted. An alternate assumption is that the Defense is able to perform accurate impact-point predictions and carry out an adaptive defensive strategy. This possibility is discussed later in Section IX.

3) The Defense enjoys perfect battle management capability in that it can launch an interceptor at any incoming object if it so desires.

4) There is no failure by the Defense to acquire and track incoming objects (there may, of course, be subsequent discrimination failures). More generally, we could assign a positive probability to this type of sensor leakage.

5) The UL decoys do not interact with the OL sensors and pass through the OL layer in full strength.

6) No incoming object is attacked by more than one interceptor in either layer.

Assumptions 1), 3), and 4) are favorable to the Defense. While perhaps not completely realistic, they will at least permit us to determine upper bounds on feasible defense system performance.

IV. PROBLEM FORMULATIONS

Proceeding from the background and assumptions just given, we can now formulate carefully several questions which arise naturally in this context. The remainder of this report will be concerned with a method for answering these questions, and illustrating the kind of answer provided.

Let us first adopt the viewpoint of the Offense. Its most fundamental question is how best to attack a given LDS. In turn, it is imperative to specify how one attack is better than another in order to have any hope of deciding which one is best. We accomplish this by associating with each attack pattern the resulting overall target survival probability P_{TS}. A method for calculating P_{TS} will be given in the next section. Hence we can say that an attack is optimal if it yields a minimum value of P_{TS}. The model to be described below is designed to determine such an optimal attack pattern.

Also of interest to the Offense is the choice of decoy as expressed by the decoy/RV exchange ratios EOL, EUL. Here EOL is the number of OL decoys that can be exchanged for one RV; EUL is the analogous figure for UL decoys. We assume no overhead penalty for such off-loading. If we attribute to the Offense the capability to deploy decoys in a variety of exchange ratios then a natural question concerns the optimal choice of ratio. Basically, are a small number of heavier decoys to be preferred to a larger number of lighter decoys? Here we assume generally that lighter decoys are easier to discriminate than heavier ones, and, for the present, ignore the possibility of saturation (of the Defense's data processing capabilities). We can (and shall) answer this question by postulating an increasing functional dependence between K_{OL} and EOL, and between K_{UL} and EUL.

Questions of interest to the Defense include various trade-offs between the two layers, such as discrimination capabilities and hardware deployments. The level of discrimination (as measured by the appropriate K-value) so as to render the use of the corresponding decoy of no value to the Offense, and the impact of poor discrimination in one layer on requirements for the other layer are examples of these questions. Also the Defense must decide the issue of an adaptive vs. subtractive OL layer. The issue is one of cost vs. survival probability: an adaptive defense (if feasible at all) costs more in terms of battle management and sensor sophistication, and it must be determined if this cost is less than the additional hardware needed to allow the subtractive OL to attain the same overall survival probability. The ultimate question for the Defense is what type of LDS to deploy. We interpret this as follows: given appropriate hardware costs, a particular threat (always assumed to be optimally allocated by the Offense), and given finally a minimum desired overall target survival probability p*, the Defense will build that

LDS whose total cost is a minimum, subject to the constraint $P_{TS} \geq p^*$.

The remainder of this report will be devoted to a presentation of methodology which permits answers to questions such as these.

V. THE BASIC OPTIMIZATION MODEL

As already observed, the fundamental problem which must be addressed is the derivation of an optimal attack against a specific LDS. We divide the solution of this problem into two parts: survival probability computations and optimization. The latter is, conceptually at least, straightforward when the requisite probabilities have been obtained.

Suppose that a given LDS involves N OL interceptors with leakage probability OLK, some number of UL interceptors with leakage probability ULK, and discrimination factors K_{OL}, K_{UL}. Also, RVs have a kill probability PK, and decoy/RV exchange ratios EOL, EUL are employed by the Offense. Suppose now that a particular target, defended by i UL interceptors, is attacked by r RVs, d UL decoys, and that there are a total of R RVs and D OL decoys deployed in the attack. We shall develop a procedure to compute functions F, where

$$\text{Prob(target survives)} = F(r,D,N,R,d,i,OLK,ULK,K_{OL},K_{UL},PK). \tag{1}$$

We might call any formula of type (1) a "Mrstik formula" after the author of [1], who gave an expression for target kill probability in the presence of both interceptors and decoys (but only for a single layer defense).

Next, if MUL denotes the maximum number of UL interceptors that will be used to defend any target, the targets may first be classified by the number of such interceptors utilized. Thus let y_i, $0 \le i \le$ MUL, denote the fraction of targets defended by i UL interceptors. Then

$$0 \le y_i \le 1,$$

$$\sum_{i=0}^{MUL} y_i = 1.$$

Now assume that the Offense further classifies the targets according to the number of RVs and the number of UL decoys assigned to that target. Let the maximum of such numbers be MRV and MULD, respectively, and let $x_{i,r,s}$ denote the fraction of targets defended by i UL interceptors, and attacked by r RVs and s UL decoys. Then the Offense must select his decision variables $\{x_{i,r,s}\}$ so that the following constraints are satisfied:

$$0 \le x_{i,r,s}, \quad 0 \le i \le MUL, \; 0 \le r \le MRV, \; 0 \le s \le MULD,$$

$$\sum_{r,s} x_{i,r,s} = y_i, \quad 0 \le i \le MUL, \tag{2}$$

$$\sum_{i,r,s} \left(r + \frac{s}{EUL}\right) x_{i,r,s} \le TL - \frac{D}{EOL \cdot NT} .$$

Here NT is the number of targets, and TL is the threat level defined in Section II. We shall state that the offensive decision variables $\{x_{i,r,s}\}$ are optimal if they obey the constraints (2) and further yield a minimum value to the function

$$P_{TS} = \sum_{i,r,s} F(r,D,N,R,s,i,OLK,ULK,K_{OL},K_{UL},PK)\,x_{i,r,s} \qquad (3)$$

The optimization problem here, namely, to minimize (3) subject to (2) is a linear program, and standard computer methods are avialable for its numerical solution.

Thus the basic problem of determining an optimal offensive deployment (for fixed exchange ratios, kill probability, and threat level) can be solved fairly routinely, once the probability functions F of (1) have been obtained. The remainder of this section is devoted to this task.*

In order to evaluate the functions F we introduce related conditional probability functions G defined as follows: *if* a target is attacked by r RVs which have penetrated the OL layer, along with d UL decoys, and is defended by i UL interceptors, we put

$$\text{Prob(target survives)} = G(r,d,i,ULK,K_{UL},PK). \qquad (4)$$

Also, since the OL defense is operating subtractively, there will be a (constant) probability p that a particular RV is attacked during the OL encounter. So we can write

$$\begin{aligned} &F(r,D,N,R,s,i,OLK,ULK,K_{OL},K_{UL},PK) \\ &= \sum_{j=0}^{r} c_j \sum_{k=0}^{j} G(r-k,s,i,ULK,K_{UL},PK)\binom{j}{k}(1-OLK)^k OLK^{j-k}, \end{aligned} \qquad (5)$$

*A trusting reader who is interested primarily in results may skip to page 24.

where $c_j = \binom{r}{j} p^j (1-p)^{r-j}$, and F and its arguments are defined in (1). Note that c_j is just the probability that j OL interceptors are fired against the r RVs.

The calculation of the OL attack probability p is obviously an important requirement, but which requires assumptions and analyses beyond our present scope. In general p can be factored as

$$p = (1-\epsilon)\cdot \mathrm{BMR}\cdot p_d,$$

where ϵ is the probability that the OL sensors fail to acquire and track the RV and produce sufficient data to implement the discrimination algorithm, p_d is the probability that this algorithm designates the RV as sufficiently threatening that the Defense will want to attack it, and BMR ("battle management reliability") is the probability that the Defense can in fact launch an interceptor at it. In keeping with assumptions 3), 4) of Section III, and our general intent to provide upper bounds for defense system performance, we shall henceforth set $\epsilon=0$ and BMR=1. We now turn to the calculation of p_d.

The OL engagement involves N OL interceptors, D OL decoys, and a total of

$$R = NT \sum_{i,r,s} r\, x_{i,r,s}$$

RVs. We introduce the order statistics

$$E_1 \le E_2 \le \text{---} \le E_D,$$

$$V_1 \le V_2 \le \text{---} \le V_R,$$

of the decoy and RV observations. Let RV* denote an arbitrary but fixed RV observation (of whatever parameter is being used to effect the discrimination). Then the probability that RV* is the k^{th} largest observation is, for $1 \le k \le N$,

$$p_k = \frac{1}{R}\left(\text{Prob}(V_{R-k+1} > E_D) + \sum_{j=1}^{\ell} \text{Prob}(E_{D-(k-j)+1} > V_{R-j+1} > E_{D-(k-j)})\right). \tag{6}$$

(The summation is ignored when k=1 and otherwise $\ell = \min(k-1,R)$. And, in terms of these probabilities p_k, we have

$$p_d = \sum_{k=1}^{N} p_k. \tag{7}$$

If a, b are integers then $\text{Prob}(E_{a+1} > V_b > E_a) = \text{Prob}(V_b > E_a) - \text{Prob}(V_b \ge E_{a+1})$, and, e.g.,

$$\text{Prob}(V_b > E_a) = \text{Prob}(E_a - V_b < 0)$$

$$= \int_{-\infty}^{0} f_{E_a} * f_{V_b}(w)\,dw \tag{8}$$

$$= \int_{-\infty}^{0} dw \int_{-\infty}^{\infty} f_{E_a}(w+y) f_{V_b}(y)\,dy,$$

where f_{E_a} (resp., f_{V_b}) is the probability density function of E_a (resp., of v_B).

The double integral over a halfplane in (8) can be reduced to a single integral over a finite interval by bringing in the specific density functions and making two changes of variable. Recall that the RV and decoy observables were assumed to obey Gaussian probability distributions with a common variance. We denote this variance by σ^2, and the respective means by m_{RV}, m_{DC}. Let $p(m,\sigma^2;x)$ be the normal density function with mean m and variance σ^2, and let $P(m,\sigma^2,x)$ be the corresponding cumulative distribution function. Then

$$f_{E_a}(x) = a\binom{D}{a}P(m_{DC},\sigma^2;x)^{a-1}(1-P(m_{DC},\sigma^2;x))^{D-a}p(m_{DC},\sigma^2;x),$$

and

$$f_{V_b}(x) = b\binom{R}{b}P(m_{RV},\sigma^2;x)^{b-1}(1-P(m_{RV},\sigma^2,x))^{R-b}p(m_{RV},\sigma^2;x)$$

are the decoy and RV observable order statistic density functions. Substituting these expressions into (8), and abbreviating $P(m_{DC},\sigma^2,x)$ to $P_{DC}(x)$, etc., yields

$$\mathrm{Prob}(V_b>E_a) = a\binom{D}{a}b\binom{R}{b}\int_{-\infty}^{0}dw\int_{-\infty}^{\infty}P_{DC}(w+y)^{a-1}(1-P_{DC}(w+y))^{D-a}p_{DC}(w+y)\cdot$$

$$P_{RV}(y)^{b-1}(1-P_{RV}(y))^{R-b}p_{RV}(y)\,dy,$$

$$= c\int_{-\infty}^{\infty} P_{RV}(u)^{b-1}(1-P_{RV}(u))^{R-b}p_{RV}(u)\int_{-\infty}^{u} P_{DC}(w)^{a-1}(1-P_{DC}(w))^{D-a}p_{DC}(w)dw\,du$$

$$= c\int_{-\infty}^{\infty} P_{RV}(u)^{b-1}(1-P_{RV}(u))^{R-b}p_{RV}(u)\int_{0}^{P_{DC}(u)} z^{a-1}(1-z)^{D-a}dz\,du$$

$$= c\int_{0}^{1} \Phi(P_{RV}^{-1}(w))w^{b-1}(1-w)^{R-b}dw, \qquad (9)$$

where we have made the substitutions

$$c = a\binom{D}{a}b\binom{R}{b},$$

$$z = P_{DC}(w),$$

$$w = P_{RV}(u),$$

and the function Φ is defined by

$$\Phi(u) = \int_{0}^{P_{DC}(u)} z^{a-1}(1-z)^{D-a}dz$$

$$= B_{P_{DC}(u)}(a,D-a+1),$$

where $B_x(\ ,\ )$ is the incomplete beta function defined by

$$B_x(\alpha,\beta) = \int_{0}^{x} z^{\alpha-1}(1-z)^{\beta-1}dz,\ 0\le x\le 1,$$

for any positve numbers α, β.

The integral in (9) can be evaluated numerically by use of a Gaussian quadrature rule of order 15-20, along with beta function approximations. This in turn permits effective computer calculation of the probabilities p_k in (6), and finally of p_d in (7).

In order to complete the specification of the objective function in (3) it remains, according to (5), to give a procedure for computing the conditional survival probabilities of (4), for then formula (5) becomes applicable. Let us abbreviate $G(r,d,i,ULK,K_{UL},PK)$ to simply G, and note first that

$$G = \sum_{k=0}^{r} \text{Prob(target survives} | \text{k RVs penetrate UL)}.$$

$$\text{Prob(k RVs penetrate UL)}$$

$$= \sum_{k=0}^{r} (1-PK)^k \text{Prob(k RVs penetrate UL)}.$$

Next we have to consider how the event "k RVs penetrate UL" can occur. We shall say that an RV "leaks" through the UL layer if the discrimination algorithm does not place its observation in one of the i largest RV and decoy observations. In this case no UL interceptor is launched against it. Therefore,

$$\text{Prob(k RVs penetrate UL)} =$$

$$\sum_{\ell=0}^{k} \text{Prob(k RVs penetrate} | \ell \text{RVs leak)Prob(}\ell \text{ RVs leak)}.$$

The left hand factors in this summation can be evaluated easily. For fixed k, ℓ, let $i^* = \min(i, r-\ell)$. Then the event "k RVs penetrate UL given that ℓ have already leaked" is the event "$r-\ell$ RVs are detected by the UL defense, are fired on by i^* UL interceptors, and $k-\ell$ survive". The probability of this event is simply the probability of having r-k successful intercepts from a salvo of i^* UL interceptors, since $(r-\ell) - (r-k) = k-\ell$. Hence we have

$$\text{Prob}(\text{k RVs penetrate UL} \mid \ell \text{RVs leak})$$

$$= \begin{cases} 0, & \text{if } r-k>i^* \\ \binom{i^*}{r-k} \text{ULK}^{i^*-(r-k)} (1-\text{ULK})^{r-k}, & \text{otherwise.} \end{cases}$$

Finally, we consider the event "ℓ RVs leak" and its probability. Recall that we are postulating an offense conservative interceptor assignment logic by the Defense, so that to say ℓ RVs leak means that exactly $r-\ell$ RV observations appear in the i largest RV + decoy observations. Thus detectability here is relative to the supply of UL interceptors. In particular, we must have $i \geq r-\ell$ or else Prob(ℓ RVs leak) = 0. Now, in terms of the order statistics $\{E_j: j=1,\ldots,d\}$ and $\{V_j: j=1,\ldots,r\}$ for the decoy and RV observables, we have

$$\text{Prob}(0 \text{ RVs leak}) = \text{Prob}(E_{d-(i-r)} < V_1),$$

$$\text{Prob}(\ell \text{ RVs leak}) = \text{Prob}(E_{d-(i-(r-\ell))} < V_\ell < E_{d-(i-(r-\ell))+1}),$$

for $0 < \ell < r$, and

$$\text{Prob}(r \text{ RVs leak}) = \text{Prob}(V_r < E_{d-(i-1)}).$$

These probabilities can be expressed in terms of multiple integrals over half-planes as in (8), and subsequently reduced to single finite integrals of the form (9).

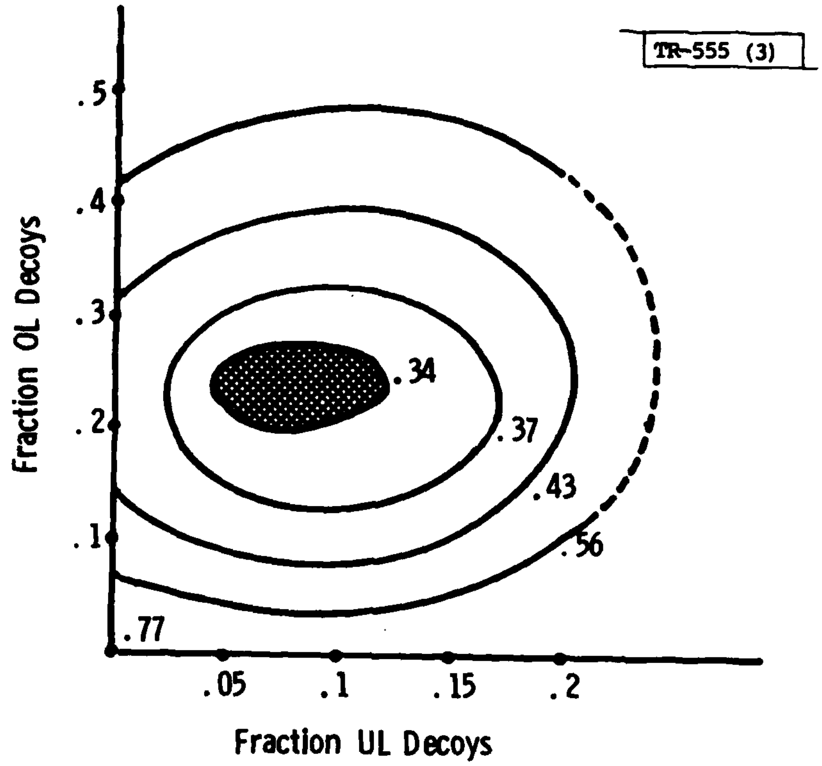
One further computational note should be made at this point about these integrals. Their value depends on the means m_{DC}, m_{RV}, and common variance σ^2 of the decoy and RV observables. The values of these parameters will further depend on the type of discriminant used (in particular, on the layer involved), but are not independent, being related to the known K-value for that layer:

$$K\sigma = m_{RV} - m_{DC}.$$

In the computer runs of this model we have arbitrarily set $\sigma=1$ and $m_{DC}=0$. As long as K remains constant, the final values of the integrals do not depend on these choices.

We have now completed the calculations required to evaluate the F-functions of (1). The utility of all this analysis and optimization to the Offense is depicted in Figs. 3 and 4. The curves displayed are curves of constant target survival probability as functions of the fractions of offensive threat level allocated to OL and UL decoys. Nominal values of relevant parameters are indicated under the curves. The only difference in parameter settings between these figures is that in the second one the OL level has been dropped to 4 (from 5) and the exchange ratios to 10 (from 20). These changes are, on balance, favorable to the Offense, as the Defense can no longer guarantee a survivor rate in excess of 30%. The primary message of such curves as these is that the quality of the Offense's attack deteriorates as the mix of decoys and RVs recedes from its optimal setting, and that the failure to deploy decoys at all results in an attack that is drastically inferior to an optimal attack with decoys.

This completes our description of the basic optimization model. It provides the essential subroutine of further programs to perform sensitivity analyses, study trade-offs, and set discrimination and deployment requirements. The next three sections are devoted to a presentation of prototypical results along these lines.



Parameters:

OL level = threat level = 5 (5000 for MM)

1 UL interceptor/target

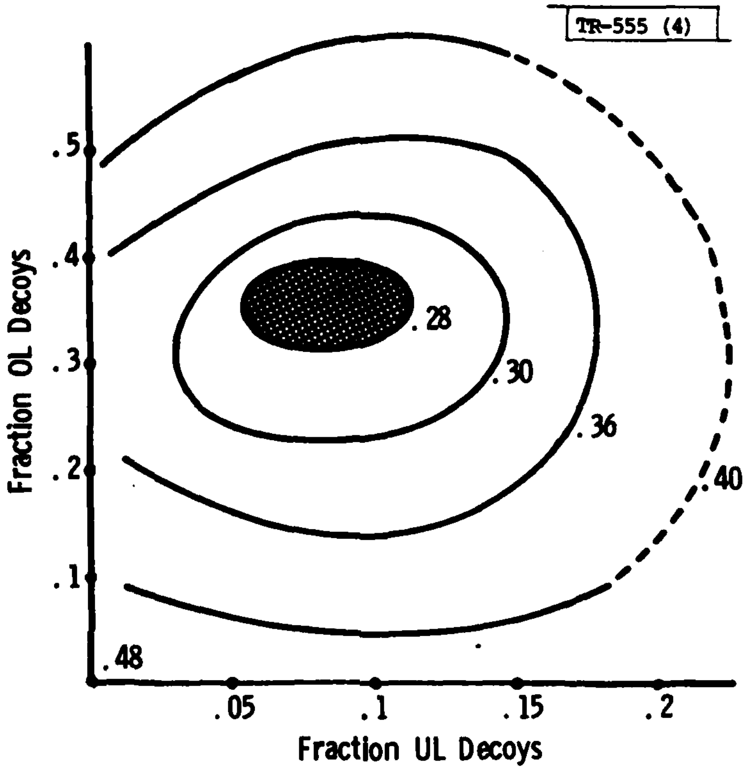
$K_{OL} = K_{UL} = 1.5$ (normalized K's = 1.15)

Exchange ratios = 20

RV reliability = 90%

OL int. reliability = 85%, UL reliability = 90%

Fig.3. Target survival probability contours.



Parameters:

OL level = 4, threat level = 5

1 UL interceptor/target

$K_{OL} = K_{UL} = 1.15$

Exchange Ratios = 10

RV Reliability = 90%

OL int. = 85%, UL int. reliability = 90%

Fig. 4. Target survival probability contours.

VI. ANATOMY OF AN OPTIMAL ATTACK

The linear optimization model described by constraints (2) and objective function (3) in the last section is insufficient of itself to completely describe an optimal attack. Further required are the total number R of RVs and D of decoys in the attack, to specify the outcome of the OL engagement. These are *further decision variables for the Offense*. Equivalently, the Offense may specify the values FRV, FULD of the fractions of the available throw-weight which are allocated to RVs and UL decoys respectively. Then

$$R = NT \cdot FRV \cdot TL,$$

$$D = EOL \cdot NT \cdot (1-FRV-FULD) \cdot TL.$$

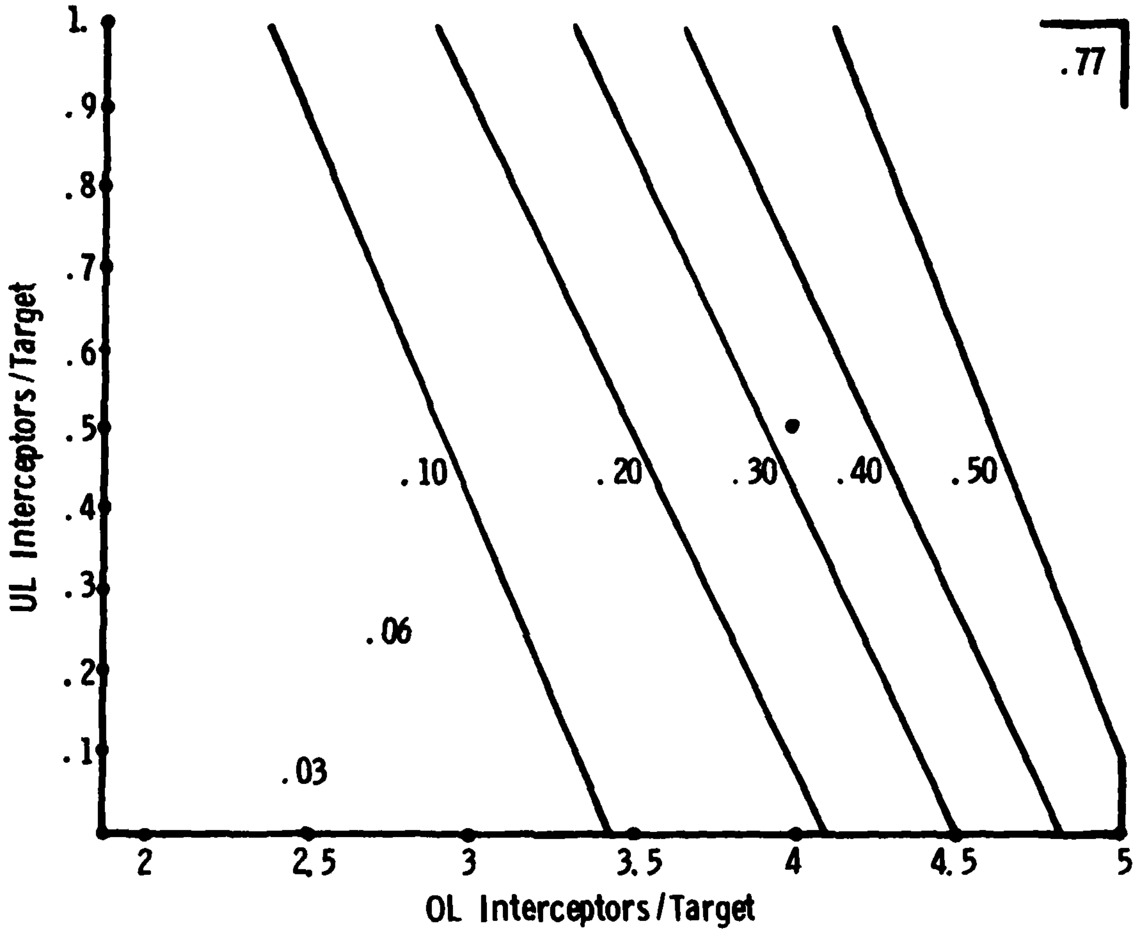
With the inclusion of these additional variables the Offense is able to completely determine an optimal attack by solving the following nonlinear program:

$$\begin{array}{c} \text{Minimize } P_{TS}(FRV,FULD) \\ \text{subject to } FRV+FULD \leq 1,\ 0 \leq FRV,FULD, \end{array} \qquad (10)$$

where $P_{TS}(FRV,FULD)$ is defined as the value of the linear program specified by (2), (3). This optimization has linearly constrained variables, hence a convex feasible set, but a nonlinear objective function. The numerical results which underlie the figures displayed hereafter in this report were derived by solving (10) using the "complex" method of nonlinear programming. This is a slow but robust search procedure which does not require gradient information. This method was originally proposed in 1965 [2] and has stood the test of time since.

As an initial illustration of the possible output of this analysis, we refer first to Fig. 5. Displayed there are again curves of constant target survival probability, this time against an all-RV attack, for varying amounts of defensive deployment of OL and UL interceptors. Suppose now that we fix attention on that scenario represented by the dot. This corresponds to an OL level of 4 and an UL deployment of 1 interceptor at every other target. The resulting target survival probability is approximately .30.

Next we permit the Offense to off-load RVs for decoys of either type at a ratio of 10 to 1 (that is, EOL=EUL=10). We also assume that the Defense can discriminate against either type of decoy at the modest level of K-factors equal to one (that is, $K_{OL}=K_{UL}=1$). Carrying out the optimization in (10) then yields the complete optimal offensive attack pattern depicted in Fig. 6. The resultant overall target survival probability here is about .12. This is, of course, very unfavorable to the Defense. It may be attributed to the low K-values in both layers, and the light UL deployment. If the K-values were increased to 1.15 and the UL deployment to 1 interceptor/target, we would revert to the situation of Fig. 4, with corresponding survival probability of about .28.



Parameters:

Threat level = 5

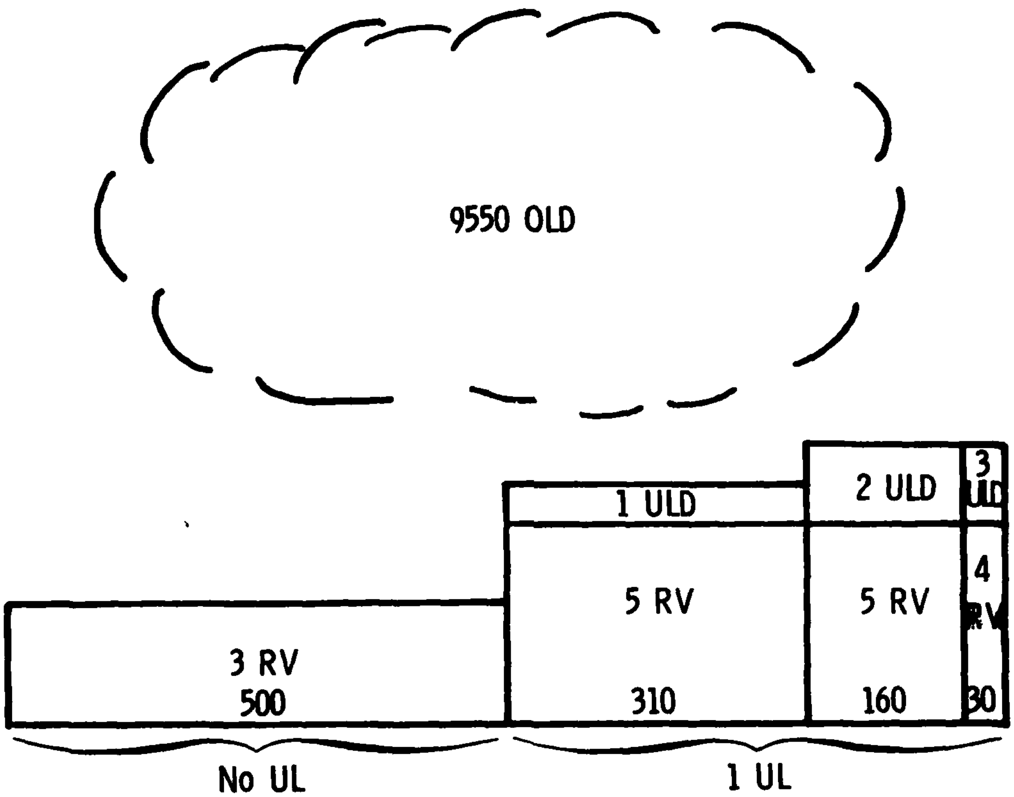
Usual reliability values

TR-555 (5)

Fig. 5. Target survival probabilities (pure RV attack).

Offense allocates 79.5% of throw-weight to RV s, 1.4% to UL decoys, and remainder to OL decoys.

On a 1000 target basis:



Parameters: See text for values

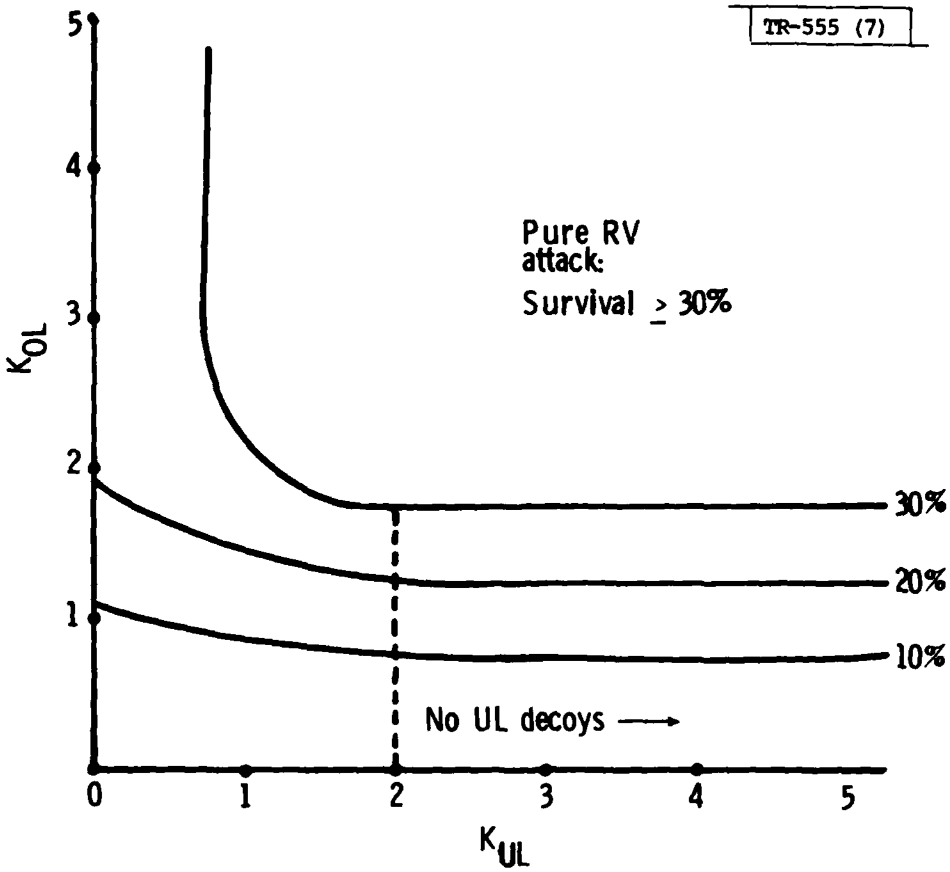
TR-555 (6)

Fig. 6. Optimal attack pattern.

VII. DISCRIMINATION REQUIREMENTS AND DECOY QUALITY

A major area of application of our optimization model is to the determination of discrimination requirements for the Defense and decoy quality for the Offense. The procedure runs as follows: holding all other parameters constant we produce a grid of target survival probabilities as functions of the two K-values K_{OL} and K_{UL}. Level curves are then interpolated by means of a bivariate cubic spline surface fitting program (which yields a finer grid of points) and a contour drawing package. The resulting plots can be subdivided into areas where, for example, no UL decoys are used or no decoys at all are used (all-RV attack). Sample output is shown in Figs. 7 and 8.

Curves such as these can be of interest to both Offense and Defense. Let us adopt the view that overall $P_{TS} \geq 30\%$ is desirable to the Defense, while overall $P_{TS} \leq 10\%$ is desirable to the Offense. Then the Defense will pay particular attention to the 30% curves; similarly for the Offense and the 10% curves. Three of the 30% curves are reproduced in Fig. 9 for decoy/RV exchange ratios of 10, 40, and 75 to 1. These curves all exhibit the same qualitative behavior, in particular, rather natural operating points for the Defense. For example, at a 40:1 ratio the Defense should attempt to achieve $K_{OL} \simeq 2.8$, $K_{UL} \simeq 1.8$. If the threat quality increases to a 75:1 ratio the Defense would be required to improve its discrimination to about $K_{OL}=3$, $K_{UL}=2.25$.



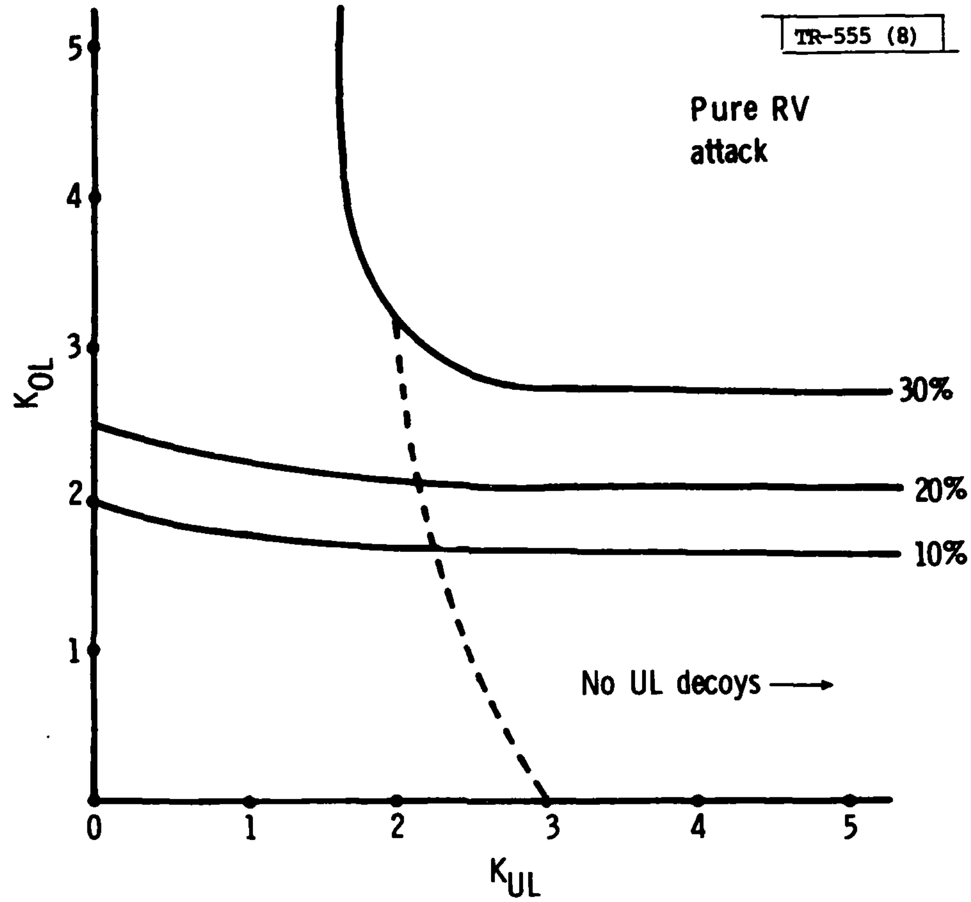
Parameters:

OL level = 4, threat level = 5

1 UL interceptor / 2 targets

Exchange ratios = 10:1

Fig. 7. Survival probability contours.



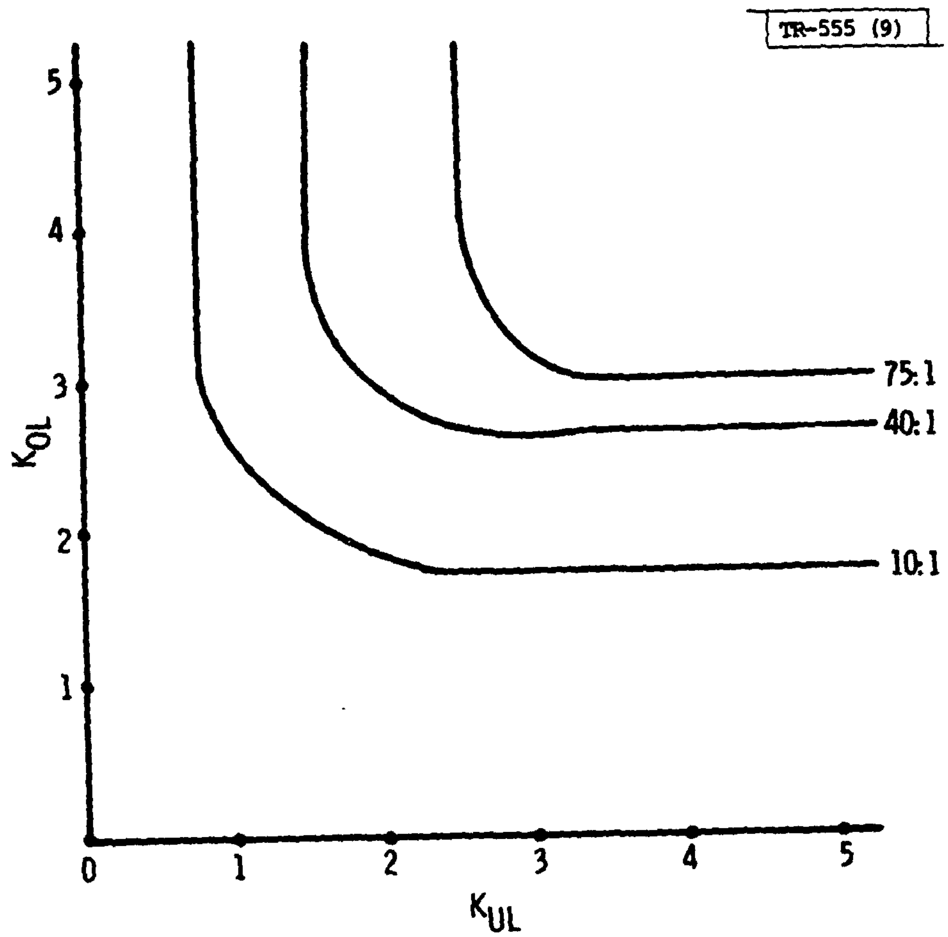
Parameters:

OL level = 4, threat level = 5

1 UL interceptor / 2 targets

Exchange ratios = 40:1

Fig. 8. Survival probability contours.



Parameters:

OL level = 4, threat level = 5

1 UL interceptor / 2 targets

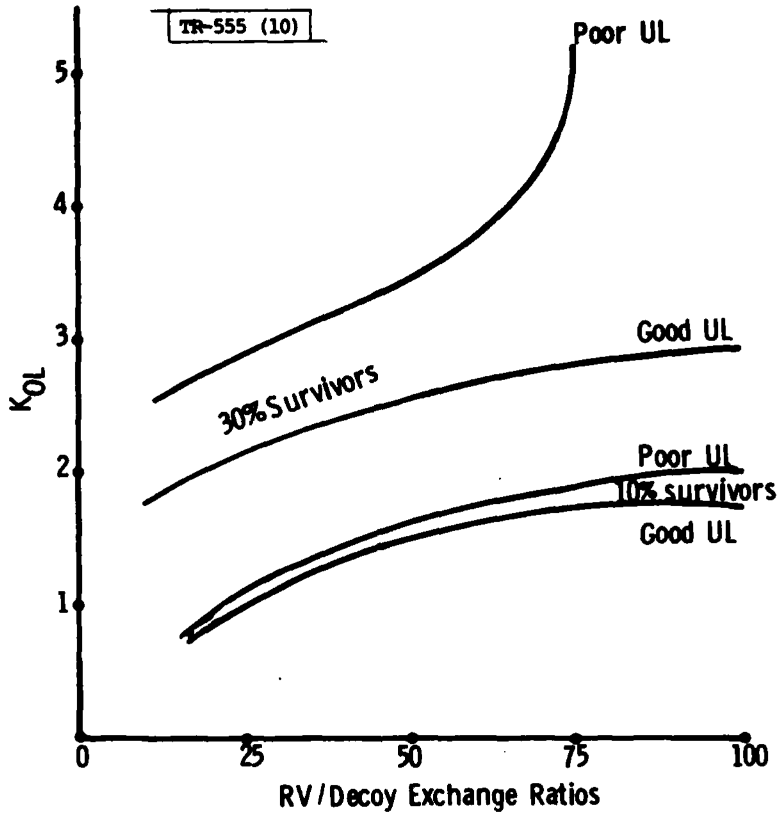
Usual reliability values

Fig. 9. 30% survival curves.

One comment should perhaps be inserted at this point concerning the interpretation of the number K_{OL}. We have referred to this as the OL discrimination constant and have assumed that it describes the ability of the Defense to discriminate RVs from OL decoys. Now in practice there may be more than one type of sensor system operating exoatmospherically. For example, there may be a combination of sensors onboard both probes and interceptor buses; if so, there will be trade-offs between the effectiveness of the two types of sensors. The present analysis does not directly lead to requirements for, say, probe sensor capability. It only stipulates that OL discrimination must collectively be of such and such a quality. If the probe alone can achieve this, no further analysis is required. If not, and if the buses are also capable of discrimination, then the issue of handover (sensor-to-sensor correlation) is encountered. However, this issue is not considered further herein.

If we now examine the 10% curves in Figs. 7 and 8 we see that the crucial factor for these parameter sets is the Offense's ability to deploy a sufficiently good OL decoy. Considering specifically Fig. 8, the Offense must be able to deploy an OL decoy with a K of at most 2. However, a slightly better OL decoy (lower K_{OL}) will permit use of a much poorer UL decoy. These observations result from the fact that the OL portion is carrying out most of the defense in this example.

Bearing this last statement in mind we offer a further application of our method. We want to study the trade-off between OL and UL discrimination requirements as a function of decoy/RV exchange ratios. We consider our usual deployment parameter settings and interceptor/RV reliabilities, and we arbitrarily say that UL discrimination is good (poor) according as $K_{UL} = 3(1.5)$. Then Fig. 10 displays K_{OL} requirements for both the 10 and 30% survival curves, as a function of exchange ratio (assumed the same for both types of decoy). We see that if the Offense can deploy a good OL decoy ($K \leq 2$) then the quality of UL discrimination is largely irrelevant; the Offense can achieve its 10% survival goal. On the other hand, for the Defense the UL quality is very important to its overall success. With a good UL it is adequate to achieve a K_{OL} value of 3. But with a poor UL, the requirement on K_{OL} is greatly increased, and no value is adequate for an exchange ratio in excess of 75:1. If this latter set of circumstances were to occur the Defense would have to recognize its deployment as being out of balance with the existing threat. There would then be two options: either improve the value of K_{UL} or else increase the amount of hardware (interceptors) deployed.



Parameters:

OL level = 4, threat level = 5

1 UL interceptor / 2 targets

Usual reliability values

Good UL: K_{UL} = 3; Poor UL: K_{UL} = 1.5

Fig. 10. OL discrimination requirements.

VIII. OPTIMAL EXCHANGE RATIOS

As a final application of our model we study the problem of an Offense which has the technical capability of off-loading RVs for both types of decoys up to more or less arbitrary levels. The natural question concerns the optimal choice of exchange ratios.

This question is not completely well posed as it stands. To remedy this defect we must first consider the effect, on the Defense, of offensive deployments at different exchange ratios. We shall subscribe to the general principle that lighter decoys are easier to discriminate than heavier ones. More precisely, we shall adopt a functional model which relates exchange ratio to K-value:

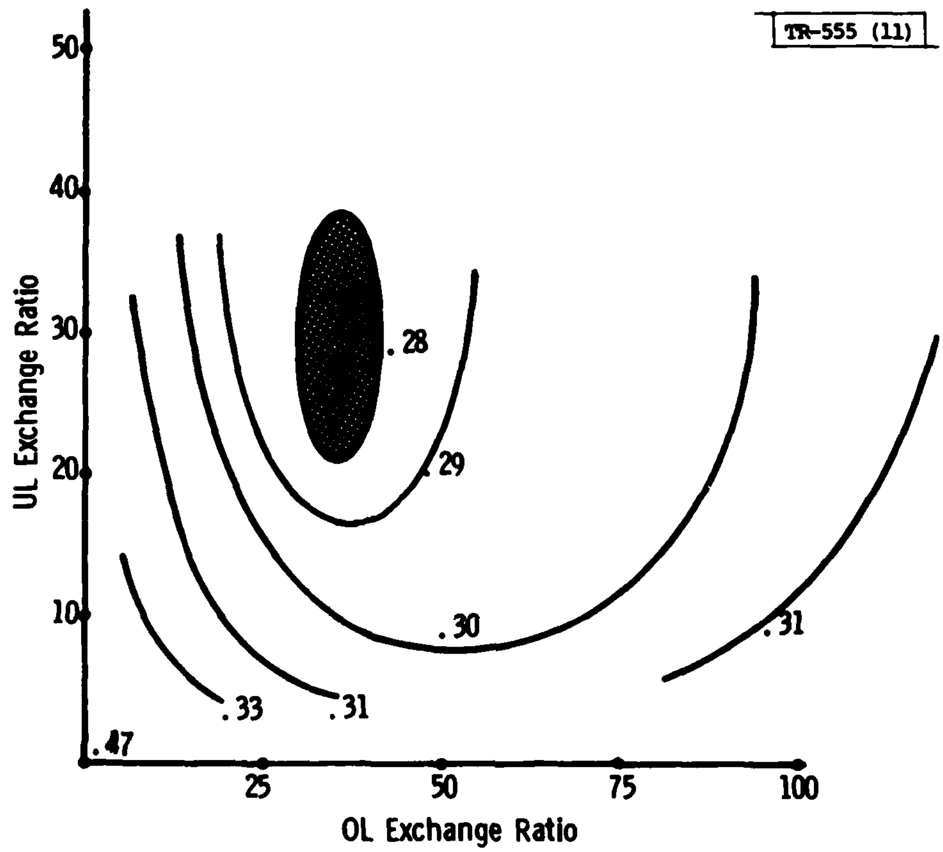
$$K = K_0 \log_{10}(\text{ex. ratio}) \qquad (11)$$

Here K_0 is often called the normalized K-value; it evidently is that value of K associated with an exchange ratio of 10.

Our question above can now be fully specified by stating that one choice of a pair of exchange ratios by the Offense is preferred over another choice if it results in a lower target survival probability figure when all other parameters are held constant. This preference ordering and hence eventual optimality are thus dependent on a particular choice of deployment parameters, reliability values, and normalized discrimination K-values.

An example of this kind of optimization is given in Fig. 11, wherein curves of constant target survival probability are displayed as functions of decoy/RV exchange ratios. Parameter settings are at their usual values, except that UL deployment has been increased to 1 interceptor per target. We see that the survival probability function whose level curves are displayed is rather flat throughout a wide region. However, the value $P_{TS} = .47$ for the all-RV attack does emphasize again the benefit accruing to the Offense for the usage of decoys. In general, we note an area of optimality where $20 \leq EUL \leq 40$, $30 \leq EOL \leq 40$. This graph indicates first, that the model (11) has largely damped out fluctuations in P_{TS} resulting from varying exchange ratios and second (and consequently), there is a wide area of acceptable exchange ratios for the Offense. It appears that the greater ability to discriminate light weight decoys is closely balanced by the need to discriminate better when faced by larger numbers of decoys. Thus precise choice of the parameters EUL, EOL is less crucial to the Offense than, say, the values of the fractions FRV, FULD, appearing in (10), and which describe how the available throw-weight is apportioned among RVs and decoys.

As always, we stress that different curves could be obtained for different parameter settings, and, in particular, for an alternate model relating discrimination to exchange ratio in place of (11).



Parameters:

OL level = 4   threat level = 5

1 UL interceptor / target

Normalized K-values (i.e., K_0's) = 1.25

Fig. 11. Survival probability contours as function of Decoy/RV exchange ratios.

IX. LDS WITH ADAPTIVE OVERLAY

Heretofore we have restricted our analysis to the case of a LDS with subtractive overlay. This was assumption 2) in Section III and was enforced initially for reasons of both mathematical and operational simplicity. By this is meant that on the one hand, the probability analysis of OL effectiveness is condensed to the evaluation of the "p" in formula (5), and on the other hand this assumption represents a less stressing mode of OL operation in that there are no impact point prediction requirements on the OL sensors and data processors.

The drawback to this assumption is, of course, that it cedes one of the prime advantages of a LDS, namely the cost leverage which results from the ability to preferentially defend only a subset of the entire target complex. Let us therefore consider this situation in more detail. Thus we specifically hypothesize sufficient D^3, tracking, and data processing capabilities on the part of the Defense so that an attack in progress can be evaluated, aimpoints determined, and an adaptive-preferential OL assignment implemented. This hypothesis immediately leads to two interesting optimization problems. First, confronted with an attack in progress, how should the Defense optimally utilize overlay? Second, knowing that the Defense will act optimally in this sense, how should the Offense best structure its attack (subject to a given throw-weight constraint)? As usual, we understand that optimality on both sides refers to overall target survival probability.

We can and shall greatly simplify the ensuing analysis by assuming that the Defense's discrimination is sufficiently good to preclude the use of decoys by the Offense. That is, we shall only discuss the case of an all-RV attack.

The Defense's deployment can now be described by a vector $y = (y_0, y_1, \ldots, y_{MUL})$ and a value OLL, where y_i is the fraction of targets defended by i UL interceptors, MUL is the maximum number of UL interceptors that are assigned to defend any particular target (so that $0 \leq y_i \leq 1$, $\sum y_i = 1$), and OLL is the OL level (= total available number of OL interceptors ÷ number of targets = N/NT in earlier notation). We also assign an integer MOL representing the maximum number of OL interceptors that can be assigned (adaptively) to defend any target.

The values of $y_0, y_1, \ldots, y_{MUL}$, OLL are considered to be fixed in what follows. In actuality, of course, they are important decision variables for the Defense. However, their explicit determination depends at least partially on hardware (interceptors, sensors, computers) costs, and so we do not consider their choice further here.

The Offense's attack can be described by a (MRV+1) × (MUL+1) matrix X, whose entries $x_{r,i}$ are the fractions of targets attacked by r RVs and defended by i UL interceptors. These are the basic decision variables for the Offense (once a threat level is specified), and must obey the constraints

$$\sum_{r} r\, x_{r,i} = TL,$$

$$\sum_{r} x_{r,i} = y_i, \qquad 0 \le i \le MUL, \tag{12}$$

$$x_{r,i} \ge 0, \qquad 0 \le r \le MR,\ 0 \le i \le MUL$$

Now, when the Defense observes the attack in progress it is able, by hypothesis, to assess the threat to each target. That is, it can determine the values of the $x_{r,i}$. It must then make an allocation of the OL defense to the various targets based on its knowledge of the entries of X. It does so by assigning some number j (possibly j=0) of OL interceptors to each target. This number will depend on the values of i and r already associated with that target. Thus the Defense must select a 3-dimensional array $Z = [z_{r,i,j}]$ of decision variables, where $z_{r,i,j}$ = fraction of targets attacked by r RVs, and defended by i UL interceptors and j OL interceptors. These variables must further satisfy the constraints

$$\sum_{r,i} j\, z_{r,i,j} = OLL,$$

$$\sum_{j} z_{r,i,j} = x_{r,i}, \qquad 0 \le r \le MRV,\ 0 \le i \le MUL, \tag{13}$$

$$z_{r,i,j} \ge 0, \quad \text{all } r,i,j.$$

We postulate that both sides will behave rationally in the selection of their respective decision variables. That is, proceeding backwards in time, the Defense will choose Z so as to maximize the overall target survival probability P_{TS} (based on its observation of X); the Offense will choose X so that this maximum value of P_{TS} is a minimum (based on its observation of the vector y); and the Defense will have selected y and OLL so that this minimum value is not less than some specified threshold (e.g., 30%) and, subject to this crucial constraint, so that its overall deployment cost is a minimum.

Note that one other important simplification is being made here. Namely, we are assuming (as before) superior battle management and handover capabilities (in addition to perfect impact-point determination), so that the Defense can indeed launch an OL interceptor against most incoming RVs. For example, if as part of its optimal determination of Z, the Defense finds $z_{3,1,2} = .2$, then 20% of its targets are defended by 1 UL interceptor and attacked by 3 RVs. The Defense now needs to attack 2 of those RVs with one OL interceptor apiece, and for our analysis we are explicitly assuming the ability to carry out such assignments.

To precisely formulate the optimizations, we need, in analogy with (3), an expression for P_{TS}. This in turn requires conditional probability functions of the form

F(r,i,j,OLK,ULK,PK)
= Prob(target survives | it is attacked by r RVs, defended by i UL and j OL interceptors),

analogous to (1). Given these probabilities, we can then write

$$P_{TS} = \sum_{r,i,j} F(r,i,j,OLK,ULK,PK)\, z_{r,i,j}. \qquad (15)$$

The linear program: max P_{TS} in (15) subject to the constraints (13), is then solved by the Defense to obtain optimal values for the z variables.

Let us denote the maximum value of P_{TS} obtained in this way by g(X). (The function g also depends on OLL and implicitly on the other y variables but at this stage we are assuming these to be fixed.) The Offense now solves the nonlinear program min g(X) subject to the constraints (12) to obtain optimal values for the x variables. It is very important to both sides that this program be solved accurately. Otherwise, in particular, the Defense might underestimate the Offense's ability to penetrate its protection and inflict damage. Fortunately, this problem does involve enough mathematical structure so that an optimization procedure can be guaranteed to converge.

The essence of this structure is that the constraint set defined by (12) is a bounded simplex in the Euclidean space of dimension (MRV+1) × (MUL+1), and the function g is concave there. Hence the minimum of g is attained at an extreme point of the simplex. Since the extreme points can be generated by solving systems of linear equations derived from the constraints in (12), it is possible for small scale problems to simply generate all extreme points and directly minimize g over this finite set. A more sophisticated approach might be to an extreme point ranking algorithm in conjunction with a linear lower bound ℓ for g (that is, ℓ is a linear function and $\ell(X) \leq g(X)$)[4].

We note that since the constraints (12) involve a total of MUL + 2 equations, of which only MUL + 1 are independent, each extreme point, and hence in particular the optimal solution, will have at most this many positive components. That is, in the matrix X which characterizes the optimal attack pattern, no more than MUL + 1 entries will be positive.

In order to actually carry out these optimizations numerically it is, of course, necessary to have assigned values to the probabilities defined in (14). For present purposes we adopt a direct and possibly somewhat simplistic approach to this assignment. That is, we ignore certain subtle issues pertaining to multiple low-level nuclear bursts and possible RV fratricide, and hence assume that the effect of several RV attacks on each target is strictly cumulative. Then by standard binomial probability analysis we have

$$F(r,i,j,OLK,ULK,PK)$$

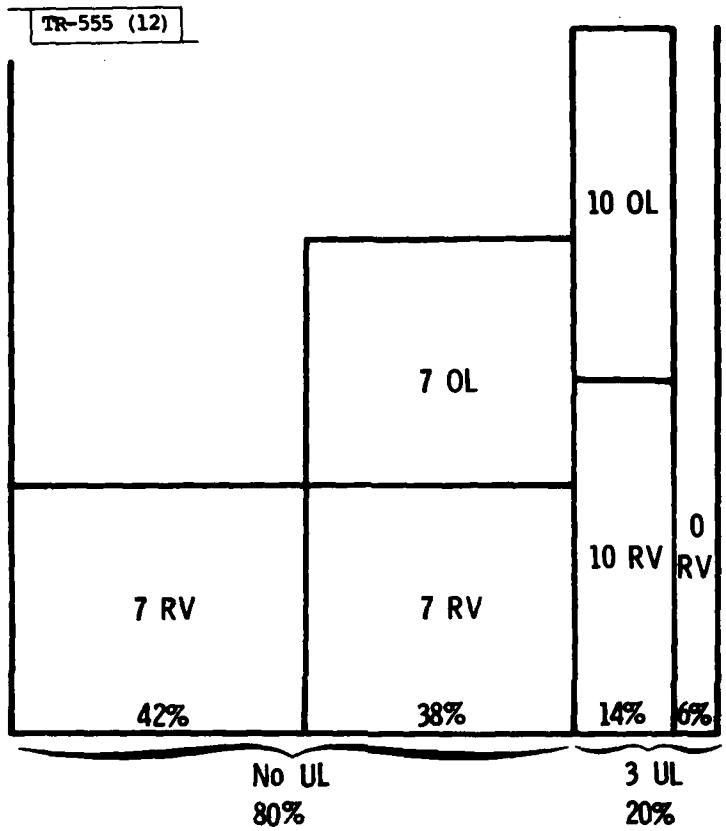
$$= \sum_{n=0}^{j}\left[\sum_{m=0}^{i}(1-PK)^{r-m-n}\binom{i}{m}(1-ULK)^{m}ULK^{i-m}\right]\binom{j}{n}(1-OLK)^{n}OLK^{j-n}$$

A computer program which implements the foregoing analysis has been written and several test cases of an all-RV attack against a LDS with adaptive overlay have been studied with its help. In fact a further level of optimization was also carried out over the Defense's hardware decision variables $y_0,\ldots,y_{MUL}$,OLL, using nominal hardware costs and the constraint that overall survival probability should not drop below 30%. Typically it was observed that the Defense would optimally leave some targets undefended with underlay, and would protect the remainder uniformly at the maximum level MUL or at one less. Then, typically, the Offense would attack the non-underlayed targets uniformly at a level of 40-70% of the maximum level MRV, and the underlayed targets at either this maximum level or not at all. Finally, the Defense typically ignores part of the attack on the non-underlayed targets and defends the remainder at the level of attack.

By way of illustration we exhibit in Fig. 12 the structure of an optimal attack and defense pattern where the basic parameters have been set as follows:

$$TL = 7,$$
$$OLL = 4,$$
$$y = (.8,0,0,.2),$$

and the usual reliability values assigned to RV and interceptor performance. Because of the OL adaptive capability the Defense does not have to match the Offense's threat level in its overlay to achieve 30% survivability. Under the assumption that the OL



Parameters:

OL level = 4, threat level = 7

20% of targets defended by 3 UL interceptors

MRV = 10, MUL = 3, MOL = 10

Usual reliability values

Fig. 12. Optimal offense-defense deployments; Pure-RV attack, adaptive overlay (P_{TS} = .3).

interceptor unit cost is 75% of the UL interceptor unit cost, the above choice of y is optimal for the Defense. If the Defense could only deploy a subtractive overlay then it would be optimal to set OLL = 6.8 and not deploy any underlay. This would increase defensive costs by about 25%. In this case the optimal offensive attack pattern is the uniform one.

In general, as various cases are studied, we observe that a defense whose overlay operates subtractively always involves a heavy, usually maximal (i.e., OLL = TL) overlay deployment, while optimal adaptive OL deployments usually range from 50 to 70% of the threat level.

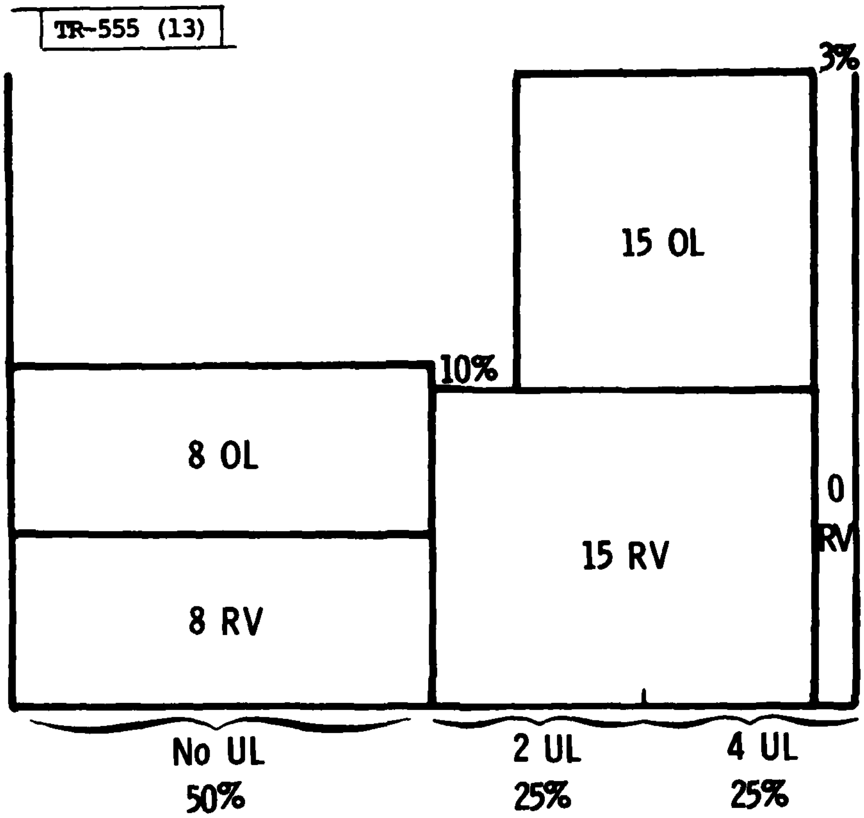
One major issue here is, naturally, the type (adaptive or subtractive) of overlay that should be deployed as part of a LDS. In general, adaptive defenses are cheaper than subtractive ones, on average by about 15%, depending on the unit costs of OL and UL interceptors for the attack levels and reliabilities considered. But of course they require a more sophisticated support system (probes, sensors, tracking algorithms, data processors and battle managers), and its integration into the national C^3 system. Thus the crucial question is whether, for specified values of the relevant parameters (OLK, TL, units costs) this more complicated support system can be developed and deployed for less than the approximate 15% saving that would accrue by use of an adaptive overlay. If not, the Defense might as well settle for the subtractive overlay. There may, of course, be other factors involved in such a decision, such as the time required to solve the more stressing technical problems associated with an adaptive OL support system, and the desirable cost leverage of an adaptive overlay to counter an increasing threat.

As a final illustration of optimal tactics we give in Fig. 13 the results of the analysis for the scenario defined by the following parameter values:

$$\begin{aligned} TL &= 11 \\ OLL &= 9.5 \\ y &= (.5,0,.25,0,.25). \end{aligned}$$

Here we have much higher threat and OL levels and two positive levels of UL deployment. In this example, we see the Defense matching the RV threat to each target up to the extent of its OL level, and the Offense attacking each underlayed target at the maximum permitted level (MRV = 15 here), up to the extent of its threat level. The resulting overall probability of survival is only .25.

Becuase of the effective (indeed, optimal) use of an OL layer that is made with adaptive capability, the question arises as to the necessary of an UL component in a LDS. Typically one expects that most of the required defense is carried out at high altitudes by OL interceptors, and that the underlay serves to cover the inevitable leakage. Whether this layer is actually needed then depends on the level of the offensive threat (TL) and the effectiveness of the OL component (OLK).



Parameters:

OL level = 9.5, threat level = 11

25% of targets defended by 2 UL, 25% by 4 UL

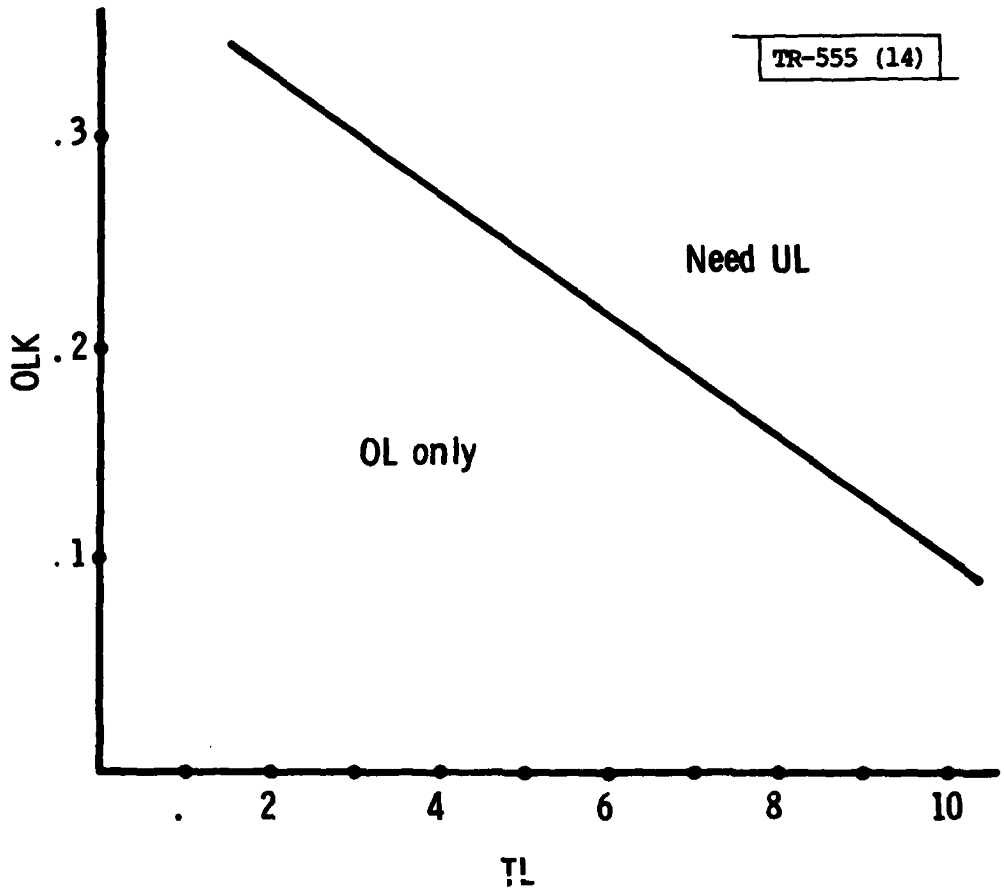
MRV = 15, MUL = 4, MOL = 15

Usual reliability values

Fig. 13. Optimal offense-defense deployments; Pure-RV attack, adaptive overlay (P_{TS} = .25).

This question can be addressed by studying P_{TS} as a function of these two variables, while treating all other variables as parameters, held constant. It is assumed that the Defense deploys an overlay-only system, with OLL = TL, and desires to maintain P_{TS} at a minimal level. When the actual survival probability from a particular encounter drops below this level (hereafter set at 30%), it is concluded that an underlay component is necessary. Fig. 14 shows a typical result: the (TL, OLK) quadrant is divided into an "OL-only" and a "need UL" partition by a line which is, in turn, constructed from several least squares lines fitted to scatter plots of TL vs. P_{TS} for different fixed values (.15,.2,.25) of OLK.

Again, we note that we have considered RV-only attacks in this section. Further work is required to determine the level of discrimination required to force the Offense to be an all RV attack.



Parameters:

MRV = 15, MUL = 0, MOL = 15

Usual reliability values

Fig. 14. Trade-off between OL leakage and threat level, when $P_{TS} = .3$.

X. SUMMARY

A layered defense system analysis model has been developed. It permits determination of overlay and underlay discrimination and deployment requirements, sensitivity studies, precise description of optimal attack and defense targeting tactics, and cost effectiveness comparisons of systems with adaptive vs. subtractive overlay.

The essence of the model is the use of a nested sequence of linear and nonlinear optimizations to produce an optimal attack against any given LDS configuration. It is this feature of optimality that gives the model its special flavor. In particular, it is to be emphasized that all results are derived rigorously and analytically, and that no Monte Carlo simulations are required or utilized.

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14 TR-555

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REPORT DOCUMENTATION PAGE

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1. REPORT NUMBER 18 ESD-TR-81-2	2. GOVT ACCESSION NO. AD-A099451	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) 6 Discrimination and Deployment Requirements For Layered Defense Systems		5. TYPE OF REPORT & PERIOD COVERED 9 Technical Report
		6. PERFORMING ORG. REPORT NUMBER Technical Report 555
7. AUTHOR(s) 10 Richard B. Holmes		8. CONTRACT OR GRANT NUMBER(s) 15 F19628-80-C-0002
9. PERFORMING ORGANIZATION NAME AND ADDRESS Lincoln Laboratory, M.I.T. P.O. Box 73 Lexington, MA 02173		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Project Element Nos. 63304A 63308A
11. CONTROLLING OFFICE NAME AND ADDRESS Ballistic Missile Defense Program Office Department of the Army 5001 Eisenhower Avenue Alexandria, VA 22333		12. REPORT DATE 11 18 Febr[redacted]81
		13. NUMBER OF PAGES 64
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Electronic Systems Division Hanscom AFB Bedford, MA 01731 12 62		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

None

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

optimal attack
survival probability
layered defense system (LDS)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A mathematical model is described which permits an analysis of optimal attack and deployment of a layered defense system. The model can also be used to determine discrimination requirements of such a system, and optimal RV/decoy exchange ratios.

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